Period 3, April 14, 2025

$$H(t) = -t^3 - 3t^2 + 288t + 1300$$

$$\int_0^{12} (-T^3 - 3T^2 + 288T + 1300)\, dT = 29424 = \#\text{ of cars}$$

Total between 0 and 12

Average value

$$\frac{1}{12}(29424) = 2452 \ \text{ve/hour}$$

Find Tangent Line at $T=12$

Set it equal 2000

$$H'(T) = -3T^2 - 6T + 288$$

$$H'(12) = -3(144) - 6(12) + 288$$

Slope $= -216$

$$H(12) = -(12)^3 - 3(12)^2 + 288(12) + 1300$$

$$= 2596$$

Point $(12, 2596)$

$$y - 2596 = -216(x-12)$$

$$L(T) = y = 2596 - 216(x-12)$$

$$2000 = \cancel{2596} - 216(x-12)$$

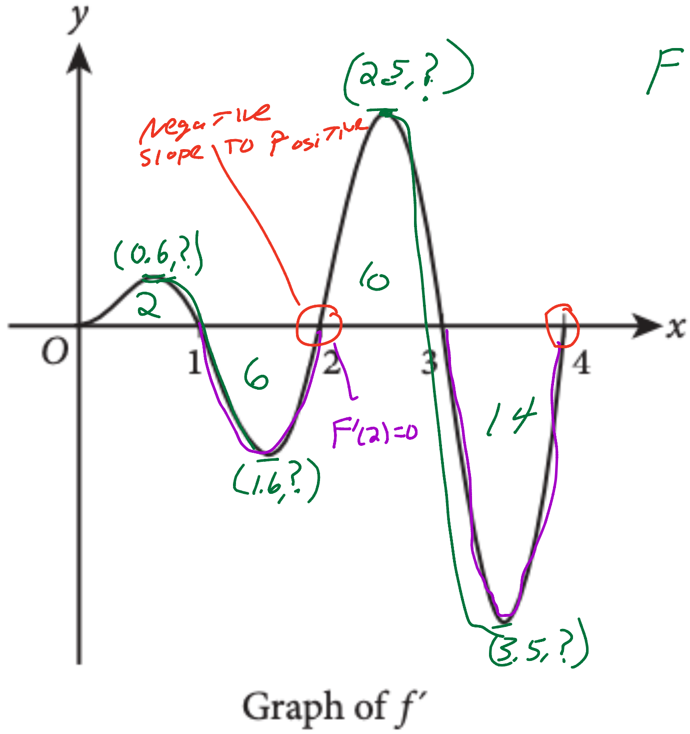
$$-2596 \quad -\cancel{2596}$$

$$\frac{-596}{-216} = \frac{-\cancel{216}(x-12)}{-\cancel{216}}$$

$$2.759 = x - 12$$

$$+12 \quad +12$$

$$14.76 = T$$



Graph of f'

$F''(2.5)=0,\ F''(3.5)=0,\ F''(0.6)=0$
$F''(1.6)=0$

$F(2)=5$

$F(0)=\left[\int_2^0 F(x)\,dx\right]+5 = +6-2+5=9$

$F(1)=\left[\int_2^1 F(x)\,dx\right]+5=6+5=11$

$F(3)=\left[\int_2^3 F(x)\,dx\right]+5=10+5=15$

$F(4)=\left[\int_2^4 F(x)\,dx\right]+5=10-14+5$
$=1$

Min $F(4)=1$

(a) Decreasing $F'(x)$ is negative $(1,2)\cup(3,4)$

Concave down $F''(x)$ is negative

$F''(x)=$ Slope of $F'(x)$ $(0.6,\ 1.2)\cup(2.5,\ 3.5)$

Both $(1,\ 1.6)\cup(3,\ 3.5)$

(b) $F(4)=1$ work is above

(c) Evaluate $\int_0^4 f(x)f'(x)\,dx$. $= \int F(x)\cdot \cancel{F'(x)}\cdot\frac{dU}{\cancel{F'(x)}}$

$U=F(x)$

$\frac{dU}{dx}=F'(x)$

$\frac{dU}{F'(x)}=dx$

$\int U\,dU=\frac{1}{2}U^2+C=\frac{1}{2}(F(x))^2+C\ \Big[_0^4$

$\frac{1}{2}[(F(4))^2]-\frac{1}{2}(F(0))^2=\frac{1}{2}(1)^2-\frac{1}{2}(9)^2=\frac{1}{2}-\frac{81}{2}$

$=\frac{-80}{2}=-40$

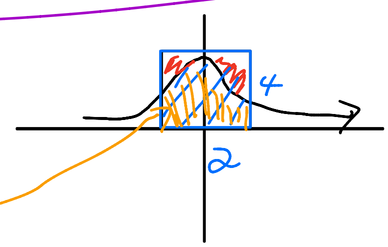
$g(x) = x^3 f(x)$.

$g'(x) = 3x^2 \cdot F(x) + x^3 \cdot F'(x)$

$g'(2) = 3(2)^2 \cdot F(2) + 2^3 \cdot F'(2) = 3 \cdot 4 \cdot 5 + 8 \cdot 0 = 60 + 0 = 60$

$\cancel{19}$, $\cancel{27}$, $\cancel{4}$, $\cancel{15}$, $\cancel{18}$, $\cancel{16}$, $\cancel{21}$, 22, $\cancel{12}$, $\cancel{13}$, 29, 30, $\cancel{8}$, 23

(19) $y = \frac{4}{1+x^2}$



$\int_{-1}^{1} \frac{4}{1+x^2}\,dx = 2\int_{0}^{1} \frac{4}{1+x^2}\,dx$

$4\arctan\frac{x}{1} \Big|_{-1}^{1}$

$4 \cdot 2 - \int_{-1}^{1} \frac{4}{1+x^2}\,dx$

$4 \cdot 2 = 8$

$4\arctan 1 - 4\arctan -1$

$4 \cdot \frac{\pi}{4} - 4\left(-\frac{\pi}{4}\right) = 2\pi$

Shaded part

$8 - 2\pi$

(B)

27.

$\int_0^1 \frac{e^x}{(2-e^x)^2}\,dx$

$u = 2 - e^x$

$\frac{du}{dx} = -e^x$

$\frac{du}{-e^x} = dx$

$\int \frac{\cancel{e^x}}{u^2} \cdot \frac{du}{-\cancel{e^x}}$

$-\int u^{-2}\,du = -\frac{1}{-1} u^{-2+1=-1} = +u^{-1} = \frac{1}{u} = \frac{1}{2-e^x}$

$\int_0^1 \frac{e^x}{(2-e^x)^2}\,dx = \frac{1}{2-e^x}\Big|_0^1$

$\frac{1}{2-e^1} - \frac{1}{2-e^0} = \frac{1}{2-e} - \frac{1}{2-1} = \frac{1}{2-e} - \frac{1}{1}$

$\frac{1}{2-e} - 1$

$\frac{1}{2-e} - \frac{2-e}{2-e}$

$\frac{1-2+e}{2-e} = \frac{e-1}{2-e}$

(D)

(a) $\lim_{h\to 0} \frac{\ln(2+h)-\ln(2)}{h} = \frac{1}{2}$

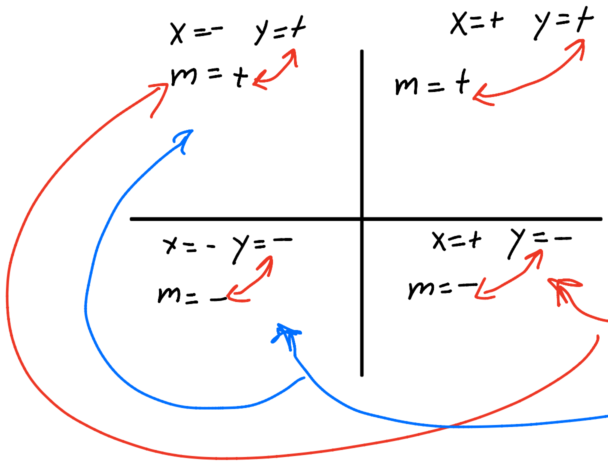
$F(x) = \ln x$

$F'(x) = \frac{1}{x}$

B

$\lim_{h\to 0} \frac{F(x+h)-F(x)}{h} = F'(x)$

$\lim_{x\to c} \frac{F(x)-F(c)}{x-c} = F'(c)$



$\frac{dy}{dx} = \frac{5}{y}$ both work

$\cancel{= -\frac{5}{x}}$

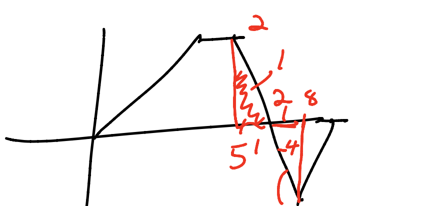
on graph Larger value of y slope close to zero

$\cancel{= \frac{x}{y}}$

$\cancel{= 5y}$ both work

12, 13, 15

(12)



$\frac{1}{2}(2)(-4) = -4$

$T=8$

$S(8)=10$

At $T=5$

$x=?$

$\int_8^5 v(T)\,dT = (4-1)+10$

$3+10$

(13)

(C)

13. $\int_{\pi/4}^{\pi/2} \sin^3\alpha\cos\alpha\,d\alpha$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{du}{\cos x} = dx$

$\int u^3 \cdot \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}} = \frac{1}{4}u^4$

$\frac{1}{4}(\sin x)^4 \Big|_{\pi/4}^{\pi/2}$

$\frac{1}{4}(\sin\frac{\pi}{2})^4 - \frac{1}{4}(\sin\frac{\pi}{4})^4$

$\frac{3}{16} = \frac{1}{4}(1)^4 - \frac{1}{4}\left(\frac{\sqrt{2}}{2}\right)^4 = \frac{1}{4} - \frac{1}{4}\cdot\frac{4}{16} = \frac{1}{4} - \frac{1}{16}$

(A)

(15)

X	F(X)
2.0	1.34
2.2	1.73

F'(2.1) = Slope between (2, 1.34) and (2.2, 1.73)

$$\frac{1.73-1.34}{2.2-2.0} = \frac{.34}{.2}$$

$(.34)\,5 = 1.7$

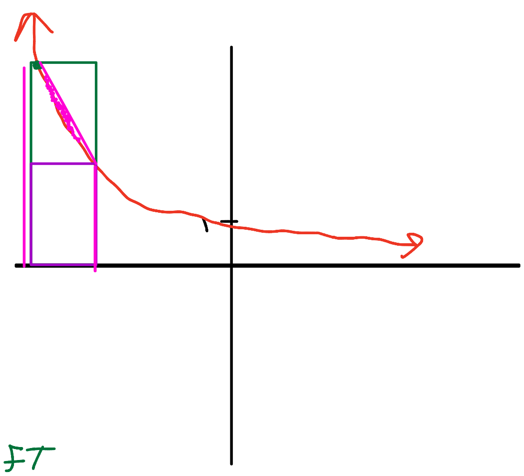
(C)

16, 18, 21

16

$$A = \int_0^1 e^{-x}\,dx = -e^{x} + C \Big]_0^1$$

$y = e^{-x}$



Left
Right
Trap

Right < A < Trap < Left

(18)

$$\int_0^6 F(x-1)\,dx = \int_{-1}^5 F(U)\,dU$$

$U = x-1$

$5 = 6-1$

$-1 = 0-1$

(B)

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(21)

$F'(x) = 2F(x)$

$F(2) = 1$

Then $F(x) =$

(A) e^{2x-4} (B) $e^{2x}+1-e^4$ (C) e^{4-2x} (d) e^{x^2-4}

$F'(x) = e^{2x-4}\cdot 2$

$\cancel{e^{2x}\cdot 2+0-0}$

$\cancel{e^{4-2x}\cdot -2}$

$\cancel{e^{x^2-4}\cdot 2x}$

~~22~~, ~~23~~, 24, 30

$$F(T) = \int_0^{T^2} \frac{1}{1+x^2}\,dx$$

$$F'(T) = \frac{1}{1+(T^2)^2}\cdot 2T$$

$$\frac{2T}{1+T^4}$$

$x = T^2$

$dx = 2T\,dT$

(D)

23. $x^3 + x\tan y = 27$ Passes Through $(3,0)$

Tangent Line Find $\frac{dy}{dx}$ at $(3,0)$

$\sec y = \frac{1}{\cos y}$

$\cos 0 = 1$

$3x^2 + 1 \cdot \tan y + x \cdot \sec^2 y \frac{dy}{dx} = 0$

$3(3)^2 + 1 \cdot \tan 0 + 3 \cdot \sec^2(0) \frac{dy}{dx} = 0$

$27 + 1 \cdot 0 + 3 \cdot 1 \frac{dy}{dx} = 0$

$3 \cdot \frac{dy}{dx} = -27$

$\frac{dy}{dx} = -9 = m$

Slope $= -9$
Point $(3,0)$

$y - 0 = -9(x-3)$

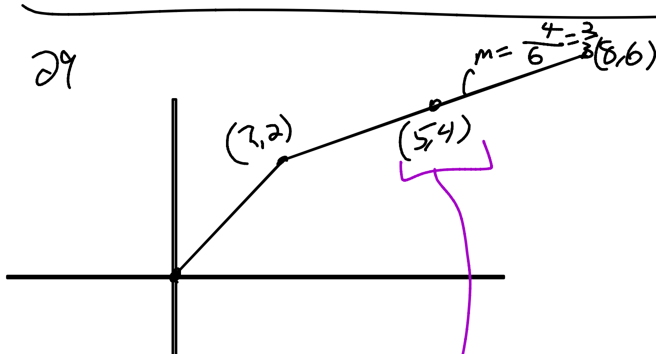
$y = -9(x-3)$

$x = 3.01$

$y = -9(.01) = -.09$

B

29



$g(4) = ?$

$F(?) = 4$

$F(5) = 4$

$g(4) = 5$

$g(x) = F^{-1}(x)$

$g'(x) = \frac{1}{F'(g(x))}$

$g'(4) = \frac{1}{F'(g(4))} = \frac{1}{F'(5)} = \frac{1}{\frac{2}{3}}$

Slope at $F(5)$

$g'(4) = \frac{3}{2}$

C

(30)

Length of interval

$$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\left(\frac{3k}{n} + 2 \right)^2 \cdot \underbrace{\frac{3}{n}}_{dx} \right)$$

Let $k = 1$ $\quad \left(\frac{3(1)}{n} + 2\right)^2 = 2^2$

$k = n$ $\quad \left(\frac{3n}{n} + 2\right)^2 = (3+2)^2 = 5^2$

A $\int_2^5 (x+2)^2 dx$

$(5+2)^2 \quad (2+2)^2$

$7^2 \quad 4^2$

B $\int_0^3 (3x+2)^2 dx$

$(3 \cdot 0 + 2)^2 = (2)^2$

$(3 \cdot 3 + 2) = (11)^2$

(c) $\int_2^5 x^2 dx$

5^2

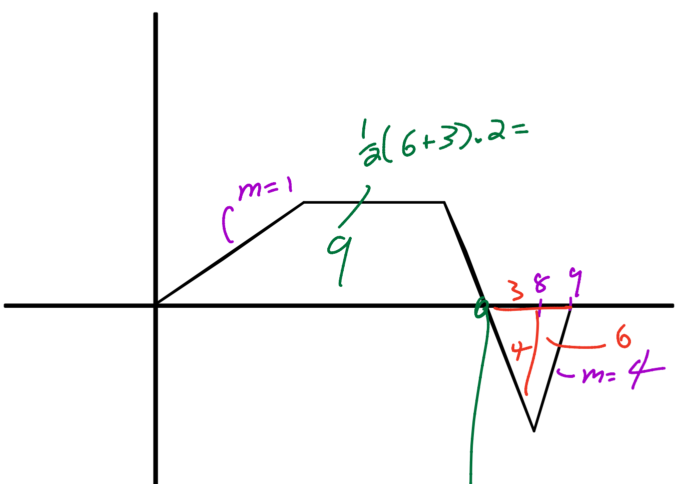
2^2

d $\int_0^3 x^2 dx$

$3^2 = 9$

$0^2 = 0$

$\frac{1}{2}(6+3) \cdot 2 =$



$S(T) = 9$

$T = 6$

10

Max acc $v'(T)$ biggest

Slope of $v(T)$

(8, 4)

11. $\int_0^6 v(T)dT = 9$

$\int_0^9 v(T)dT = 9 - 6$

$\int_9^0 v(T)dT = +6 - 9$